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Appendix 1: Zonally averaged mass stream function

The zonally averaged mass stream function is given by:

$$\psi(\phi, p) = \frac{2\pi a cos\phi}{g}\int_{P}^{P_s} \bar{v}dp$$

where φ is latitude, p is pressure, $\bar{v}$ is the zonal mean meridional wind, a is the radius of the earth and g is the gravitational acceleration.

Appendix 2: Zonally asymmetric diagnostics of the Hadley cell

In the isobaric vertical coordinate system, the spherical version of the continuity equation is given by:

$$\frac{1}{a\cos\phi}\frac{\partial u}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\cos\phi) + \frac{\partial\omega}{\partial p} = 0,$$ **Equation A1**

where u and v are the zonal and meridional components of the divergent ageostrophic flow, respectively, ω is the vertical component of the velocity, a is the radius of the earth, λ is the longitude and ϕ is the latitude.

In this section we partition the isobaric continuity Equation A1.[1,2] On each isobaric surface, the divergent, irrotational flow in Equation A1 may be written in terms of a velocity potential χ, such that:

$$u = \frac{1}{a\cos\phi}\frac{\partial\chi}{\partial\lambda\varphi}$$

and

$$v = \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\chi\cos\phi)$$ **Equation A2**

As

$$\nabla\times\nabla\chi = 0,$$

Let

$$\chi \equiv \frac{\partial\mu}{\partial p}$$ **Equation A3**

where μ is a potential function, and then combine this with Equation A1, to produce a Poison's equation in μ, i.e.:

$$\frac{1}{a^2 cos^2\phi}\frac{\partial^2\mu}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial\mu}{\partial\partial\phi} = -\omega\right)$$ **Equation A4**

which is solved numerically, given suitable boundary conditions.[1,2]

Now define a vector stream function $\vec{\psi}$ (here the over arrow indicates that this quantity is a vector), such that:

$$\psi_\lambda = -\frac{1}{a\cos\phi}\frac{\partial\mu}{\partial\lambda}$$

and

$$\psi_\phi = -\frac{1}{a}\frac{\partial\mu}{\partial\phi}$$ **Equation A5**

and taking the divergence of $\vec{\psi}$ and using Equation A4, we obtain:

$$\frac{1}{a\cos\phi}\frac{\partial\psi_\lambda}{\partial\lambda} + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\psi_\phi\cos\phi\right) = \omega$$ **Equation A6**

Clearly, from Equation A6, vertical motion ω is partitioned into two orthogonal directions so that:

$$\omega_\lambda\cos\phi = \frac{1}{a}\frac{\partial\psi_\lambda}{\partial\lambda}$$ **Equation A7**

$$\omega_\phi\cos\phi = \frac{1}{a}\frac{\partial}{\partial\phi}\left(\psi_\phi\cos\phi\right)$$ **Equation A8**

with

$$\omega = \omega_\lambda + \omega_\phi$$

It follows then from Equations A2 and A3 that the zonal component of the ageostrophic flow may be written in terms of zonal component of the ψ vector:

$$u = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left(\frac{\partial\mu}{\partial p}\right) = \frac{\partial}{\partial p}\left(\frac{1}{a\cos\phi}\frac{\partial\mu}{\partial\lambda}\right) = -\frac{\partial\psi_\lambda}{\partial p}$$ **Equation A9**

Similarly, the meridional component of the ageostrophic flow may be written in terms of meridional component of the ψ vector as follows:

$$v = -\frac{\partial\psi_\phi}{\partial p}$$ **Equation A10**

Using Equations A7 and A9 and the fact that partial derivatives are interchangeable, we have:

$$\frac{1}{a}\frac{\partial u}{\partial\lambda} + \frac{\partial}{\partial p}(\omega_\lambda\cos\phi) = 0$$ **Equation A11**

and similarly

$$\frac{1}{a}\frac{\partial}{\partial\phi}(v\cos\phi) + \frac{\partial}{\partial p}\left(\omega_\phi\cos\phi\right) = 0$$ **Equation A12**

References

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