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Boeyens JCA, Thackeray JF. Number theory and the unity of science. S Afr J Sci. 2014;110(11/12), Art. #a0084, 2 pages. http://dx.doi. org/10.1590/sajs.2014/a0084 Within recent millennia, sentient representatives of the species *Homo sapiens* have explored science with a sense of curiosity. Currently there are schoolchildren, university students and academic researchers, in Africa and elsewhere, asking questions about relativity, mass, space, particles, waves, space-time and the nature of constants in the fields of mathematics, physics, chemistry and biology.¹ Recently, questions have been raised about whether an irrational mathematical constant – designated by the Greek symbol Φ with a value of about 1.618 – can be related to a biological species constant (T), based on morphometric analyses of modern mammalian skulls, and explored in the context of probabilities of conspecificity of Plio-Pleistocene hominin fossils.²⁻⁴ We suggest that there is a strong case that this so-called 'Golden Ratio' (1.61803...) can be related not only to aspects of mathematics but also to physics, chemistry, biology and the topology of space-time.¹

A convincing case for assuming a cosmic character of the Golden Ratio can be made based on the ubiquity of logarithmic spirals. Spectacular examples include the Whirlpool Galaxy (M51), ammonites, the shape of *Nautilus* shells, Hurricane Katrina and the distribution of planets, moons, asteroids and rings in the solar system (Figure 1). The logarithmic spiral is firmly related to the Fibonacci series and the Golden Ratio of number theory. A familiar aspect of Fibonacci spirals is the way they feature in botanical phyllotaxis, the shape of kudu (*Tragelaphus strepsiceros*) horns and the curvature of elephant tusks. Less well known is the way in which the crystallographic structure of DNA, stress patterns in nanomaterials, the stability of atomic nuclides and the periodicity of atomic matter depend on the Golden Ratio.¹ Apart from the Golden Ratio, a second common factor among this variety of structures is that they all represent spontaneous growth patterns. The argument that this amazing consilience ('self-similarity') arises from a response to a common environmental constraint, which can only be an intrinsic feature of curved space-time, is compelling.¹



Sources: (left to right) NASA and The Hubble Heritage Team (Wikimedia Commons); Chris 73 (Wikimedia Commons); US National Oceanic and Atmospheric Administration (Wikimedia Commons); kongsky (FreeDigitalPhotos.net)

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Figure 1: Examples of logarithmic spirals found in nature: (from left to right) the Whirlpool Galaxy, a *Nautilus* shell, Hurricane Katrina and an ammonite.

In the context of biology, Thackeray³ has identified what he claims to be a species constant, based on the log-transformed standard error of the m-coefficient (log se_m), in regression analysis of cranial and other measurements from pairs of specimens of extant biological species (vertebrates and invertebrates), associated with regression equations of the form y = mx + c, where *m* is the slope and *c* is the intercept, using measurements of specimen A (x-axis) and specimen B of the same species (y-axis), or vice versa. Remarkably, an absolute mean log se_m value of ca 1.61 has been obtained for conspecific pairwise comparisons of extant vertebrates (mammals, birds, reptiles) and invertebrates (Coleoptera and Lepidoptera).³

An almost identical absolute mean log se_m value has been obtained from pairwise comparisons of conspecific Plio-Pleistocene hominin crania representing *Australopithecus*, *Paranthropus* or early *Homo*.⁵

An absolute mean log se_m value of 1.61 for pairwise comparison of dental measurements of hominin species has been calculated by Dykes⁶. In modern primates such as humans, chimpanzees, gorillas, orangutans and *Colobus* monkeys, the absolute mean log se_m value for pairwise conspecific comparisons of crania is approximately 1.6.⁷ The empirical evidence for a biological constant with central tendency of an absolute value of 1.61 is strong.

Transformation of the equations for relativistic quantum theory from curved space-time to Euclidean space coordinates was derived by Veblen and Hoffmann⁸ without realising the significance of the factor $\sqrt{5} = \Phi - 1/2$ that correlates the electromagnetic potentials in the underlying curved space-time and tangent space. Examined more closely, the curvature of a Fibonacci spiral, $\pi/2(2\sqrt{2}) \simeq \sqrt{5}/2$, hence constitutes a convincing measure of space-time curvature.

Also demonstrated at an early date by Harkins⁹ was the fact that stable nuclides occur in a limited region defined by convergence to the proton:neutron ratio (*p*/*n*) from unity to a value of 0.62, later interpreted¹⁰ as $p/n = 1 \rightarrow \tau$, where $\tau = 1/\Phi$. By the same reasoning, the remarkable observation that the structure of the periodic table of the elements is a function of environmental pressure¹¹ can now be accounted for in detail, as a response to space-time curvature. The stability of atomic nuclides varies from p/n = 1 at a black-hole singularity to p/n = 0.58 in Euclidean space.

The time has come to recognise that relativity and quantum theories can be integrated, and linked numerically to the value of a mathematical constant – whether in the context of space-time¹ or biology³.

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