Online supplementary material to: Bansilal S. A Rasch analysis of a Grade 12 test written by mathematics teachers. S Afr J Sci. 2015;111(5/6), Art. #2014-0098, 9 pages. http://dx.doi.org/10.17159/sajs.2015/20140098

Appendix 1: The test items

1. Solve for *x*, correct to TWO decimal places, where necessary:

1.1 $2x^2 + 3x - 7 = 0$ (4)

1.3 $7x^2 + 18x - 9 > 0$ (4)

2. The sequence 3; 9; 17; 27; … is a quadratic sequence:

2.1 Write down the next term (1)

2.2 Determine an expression for the n^{th} term of the sequence (4)

2.3 What is the value of the first term of the sequence that is greater than 269? (4)

3. Consider the function $f(x) = \frac{3}{x-1} - 2$

3.1 Write down the equations of the asymptotes of *f* (2)

3.2 Calculate the intercepts of the graph of *f* with the axes (3)

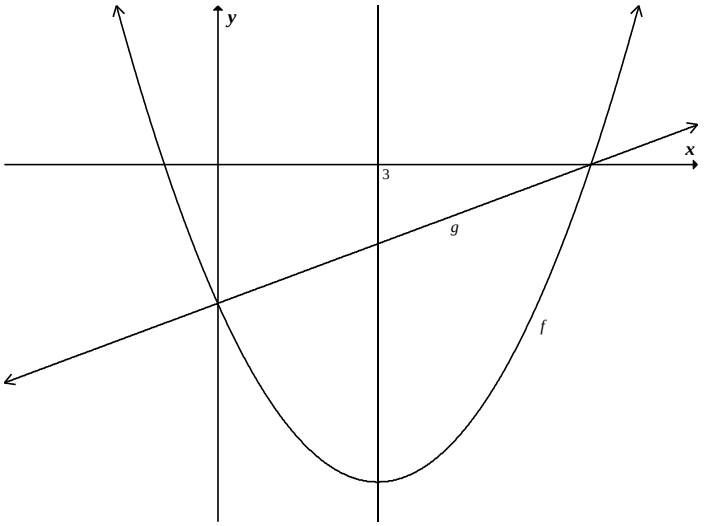
3.3 [omitted]

3.4 Sketch the graph of *f* on DIAGRAM SHEET 1 (3)

3.5 Write down the range of $y = -f(x)$ (1)

3.6 Describe, in words, the transformation of *f* to *g* if $g(x) = \frac{-3}{x+1} - 2$ (2)

4. A parabola *f* intersects the *x*-axis at B and C and the *y*-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



4.1 Show that the coordinates of C are (7; 0) (1)

4.2 Calculate the *x*-coordinate of B (1)

4.3 Determine the equation of *f* in the form $y = a(x-p)^2 + q$ (6)

4.4 Write down the equation of the graph of *h*, the reflection of *f* in the *x*-axis (1)

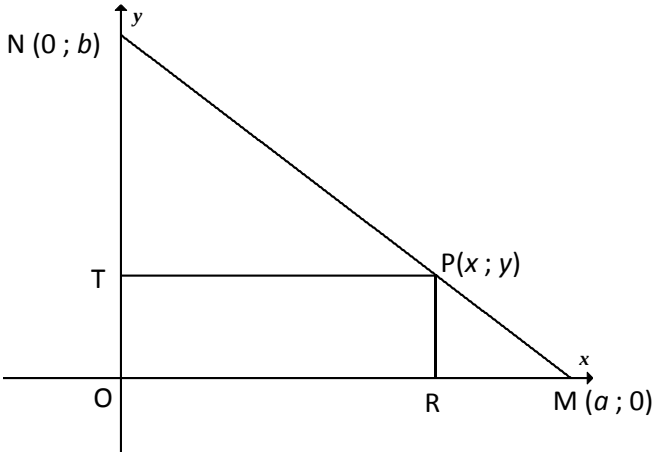
4.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$ (2)

4.6 Solve for *x* if $f(x^2 - 2) = 0$ (4)

5.1 Calculate $D_x\left[4 - \frac{4}{x^3} - \frac{1}{x^4}\right]$ (3)

5.2 Determine $\frac{dy}{dx}$ if $y = \left(1 + \sqrt{x}\right)^2$ (3)

6. A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = *a*, ON = *b* and P(*x* ; *y*) be any point on MN.



6.1 Determine an equation of MN in terms of *a* and *b* (2)

6.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN (6)

7. While preparing for the 2010 Soccer World Cup, a group of investors decided to build a guesthouse with single and double bedrooms to hire out to visitors. They came up with the following constraints for the guesthouse:

- There must be at least one single bedroom.
- They intend to build at least 10 bedrooms altogether, but not more than 15.
- Furthermore, the number of double bedrooms must be at least twice the number of single bedrooms.
- There should not be more than 12 double bedrooms.

Let the number of single bedrooms be *x* and the number of double bedrooms be *y*.

7.1 Write down the constraints as a system of inequalities (6)

7.2 Represent the system of constraints on the graph paper provided on DIAGRAM SHEET 2. Indicate the feasible region by means of shading. (7)

7.3 According to these constraints, could the guesthouse have 5 single bedrooms and 8 double bedrooms? Motivate your answer. (2)

7.4 The rental for a single bedroom is R600 per night and R900 per night for a double bedroom. How many rooms of each type of bedroom should the contractors build so that the guesthouse produces the largest income per night? Use a search line to determine your answer and assume that all bedrooms in the guesthouse are fully occupied. (3)

Appendix 2: Summary of the process of rescoring

Item	Item with comments	Fit residual (FR)	Decision run 1	Results of rescoring 1 on FR	New score/ comment
1	1.1 (4) Under discrimination; disordered thresholds	2.123	Rescore 1,2-1 3,4-2	FR=1.287 Categories still not working optimally	Maximum of 2 marks
2	1.3 (4) Slight under discrimination; disordered thresholds	1.344	Rescore 1,2,-1 3,4-2	0.120 Categories working well	2 marks
3	2.1 (1) Item characteristic curve shows haphazard fit	0.052	Leave as is	0.856 Although no rescoring, FR has changed	Left as is
4	2.2 (4) Slight over discrimination; disordered thresholds	-1.008	Rescore 0,1-0 2,3-1 4-2	-1.453 Categories still not working well	2 marks
5	2.3 (4) Slight over discrimination; disordered thresholds; categories 1,3 not working	-2.164	Rescore 1,2-1 3,4-2	-3.162 Now it is beyond reasonable limits for fit residuals	Rescore 1,2,3-1 4-2 FR=-0.928 Rescored to 2 marks
6	3.1 (2) Haphazard fit; disordered thresholds Category 1 not working	-0.851	Rescore 1,2-1	-0.692 Still has haphazard fit	1 mark
7	3.2 (3) Haphazard fit; disordered thresholds; categories 1,2 not working	-1.077 Item characteristic curve similar to Item 3.1, but less severe	Rescore 1,2-1 3-2	-0.241 Still has haphazard fit; category 1 not working	2 marks

Item	Item with comments	Fit residual (FR)	Decision run 1	Results of rescoring 1 on FR	New score/ comment
8	3.4 (3) Haphazard fit; disordered threshold; categories 1,2 not working	-1.424 Item characteristic curve similar to 3.1,3.2	Rescore 1,2-1 3-2	-1.391	2 marks
9	3.5 (1) Classic over discrimination	-1.284	Leave as is	-0.997 Item characteristic curve shows better empirical fit	Left as is
10	3.6 (2) Small distance between thresholds 1 and 2; slight haphazard fit	-0.047	Leave as is, cannot justify rescoring	0.734	Left as is
11	4.1 (1) Haphazard fit	-0.051	Leave as is	0.806	Left as is
12	4.2 (1) Over discrimination	-1.054	Leave as is	-0.0115	Left as is
13	4.3 (6) Disordered thresholds, slight under discrimination	0.621	Rescore 1,2-13-2 4,5-3 6-4	0.084 FR has improved but now greater under discrimination	4 marks
14	4.4 (1) Over discrimination	-3.219	Leave as is	-1.5990 Without any rescoring the FR has improved as a result of changes to other items	Left as is
15	4.5(2) Disordered thresholds; slight over discrimination	-1.613	Rescore 0,1-0 2-1	-1.829	1 mark
16	4.6 (4) Disordered thresholds good fit, FR=0.01	0.010	Rescore 1,2,3-1 4-2	0.590	2 marks

Item	Item with comments	Fit residual (FR)	Decision run 1	Results of rescoring 1 on FR	New score /comment
17	5.2 (3) Slight under discrimination; disordered thresholds; categories 1 and 2 not working	0.313	Rescore 1,2-1 3-2	1.049 Category 1 still not working	2 marks
18	5.3 (3) Haphazard fit; disordered thresholds	1.163	Rescore 0,1-0 2,3-1	-0.075	1 mark
19	6.1 (2) Disordered thresholds; under discrimination	2.612	Rescore 1,2-1	2.117	1 mark
20	6.2 (6) Slight under discrimination; disordered thresholds	2.372	Rescore 0,1,2-0 3,4-1 5-2 6-3	1.352	3 marks
21	7.1 (6) Adequate fit; slight under discrimination; disordered thresholds	0.386	Rescore 0,1-0 2,3-1 5,6-2	-1.155 Poor empirical fit to item characteristic curve	2 marks
22	7.2 (7) Good fit; disordered thresholds	0.084	Rescore 1,2-1 3,4,5-2 6,7-3	0.492 More haphazard fit	3 marks
23	7.3 (2) Disordered thresholds; shows differential item functioning by race and qualification; haphazard fit	1.321		Deleted because of differential item functioning	Deleted
24	7.4 (3) Disordered thresholds; over discrimination	3.454	Rescore 1-1 2,3-2	-1.854 Empirical distribution fits well	2 marks
	Total score = 75			**Rescored total = 42**	