Mathematics and economics: We should expect better models

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Mathematics is central to the modern discipline of economics, although there has always been some dispute about the value of this relationship. Leijonhufvud[1], in his satire of the economics profession, declared that the 'Math-Econ'(mathematical economists) appeared to be the high priests of this tribe. There was debate on whether their work was 'best to be regarded as religious, folklore and mythology, philosophical and 'scientific', or as sports and games'[1(p.334)].

In this commentary I want to consider whether the role of mathematics goes beyond 'sports and games' in economics and reflect on some of the ways in which mathematics is used in economics in general and in South African economics in particular.

Mathematics in microeconomic theory

Economic theory falls into two broad domains: microeconomics, concerned largely with the interactions of individuals and firms, and macroeconomics, concerned with the aggregate behaviour of economies. The dominant paradigm in microeconomics has been 'neoclassical' price theory culminating in the theory of general equilibrium. The approach is axiomatic: begin with assumptions about the key drivers of individual economic behaviour and then deduce the implications for aggregate outcomes. Two of the core assumptions underpinning this approach are:

1. Individuals have relatively fixed likes and dislikes ('preferences') which can be modelled by a 'utility function'.
2. Individuals are rational in the sense that they will act so as to realise their most preferred outcomes within the means available to them.

The mathematical machinery deployed is that of constrained optimisation with a dash of topology. In applying the framework to various specific markets, a number of innovations have been made. In many markets (e.g. agriculture and finance), the key issue is to model uncertainty. How do you decide what to plant or at what rate to sell if you need to foresee the state of the economy in the future? The approach is to marry the utility function to a probability distribution capturing the probability of different outcomes. The idea is that decisions will be governed by the expected value of the different outcomes. The framework for this approach was laid down by the mathematician John von Neumann and the economist Oskar Morgenstern and is hence referred to as 'von Neumann–Morgenstern utility theory'. It underpins the pricing models used in finance including the Black–Scholes equation. This is an intriguing case because here we see an economic theory feeding back into actual behaviour in markets that the theory is intended to describe.

The von Neumann–Morgenstern model of expected utility was first presented as an aside in their development of game theory.[2] The core purpose of that theory is to analyse strategic interactions, i.e. where the outcome achieved depends on the choices of all the agents involved. In these situations, agents need to foresee how their opponents are likely to react. One of the first applications of that theory outside economics was in modelling interactions between the USA and the Soviet Union in nuclear deterrence. Since then game theory has found applications in many different domains, and has even entered the general discourse. Concepts such as 'prisoner's dilemma' or a 'zero-sum game' have been applied to many negotiation contexts. Game theory has been particularly successful in elucidating the evolution of conflict and cooperation, even in the case of biology. Many fascinating interactions such as parent–child conflicts have been successfully modelled. From biology the concept of an 'evolutionarily stable strategy' has filtered back into economics.

One of the key areas in which this theory has been applied is in the study of 'institutions'.[3] The first economists were acutely aware that the economic well-being of countries depended on the right sort of laws and non-predatory behaviour by the state. During the elaboration of the neoclassical theory of general equilibrium, a concern with political institutions, legal frameworks and the norms underpinning market exchange were put to one side. More recently, these concerns have re-emerged as central to the performance of countries and regions.[4] Countries which have institutions that entrench predatory practices have much weaker growth performance than countries with more inclusive practices.

While modern microeconomics has a far richer understanding of the ways in which individuals interact there are still a number of continuities between the neoclassical price theory and some of the modern game theoretic models:

- The 'rational actor' model is common to both.
- Both tend to concentrate on predicting 'equilibrium' behaviour.
- The preference structures which underlie the behaviour are not themselves the subject of theoretical investigation – they are assumed to be pre-given and relatively unchanging.

All of these elements have come under scrutiny recently. The 'behavioural economics' research programme has investigated how individuals actually behave instead of starting from choice axioms describing how rational agents ought to choose.[5] This research has provided a number of robust insights into what motivates people. Firstly, many of these experiments show that human beings have a very poor intuitive grasp of probabilities, thus casting doubt on the ability of the 'expected utility' framework to adequately describe choice in uncertain conditions. Secondly, in laboratory experiments of various game theoretic interactions, such as the 'prisoner's dilemma' and the 'dictator' and 'ultimatum' games, it has been routinely found that individuals are more cooperative than the theory suggests they should be. Interestingly, students trained in economics tend to conform to the 'rational agent' model more closely than most people, suggesting once more that economic theory has the potential of influencing the actual behaviour of the agents it is supposed to describe.

A number of 'evolutionary' game theoretic models have explicitly considered less than perfectly rational agents learning from experience. These models also consider behaviour away from equilibrium. The resulting dynamics can be complicated and are typically simulated and not analytically solved out. These 'agent-based models' raise additional questions, because the space of all possible outcomes is obviously sampled rather than systematically described. Furthermore, the learning process is invariably local – it involves copying 'successful' behaviours from neighbours. It therefore differs from some of the theories to be discussed below which insist that agents should be able to learn some of the 'global' properties of the system in which they are embedded.

Finally a number of authors have begun to embed an analysis of human preferences and human behaviour in a broader account of the evolution of humanity.[6] This work draws attention to the culture in which agents operate and shows how this influences the types of behaviours that are possible.

Mathematics in macroeconomic theory

Macroeconomics has traditionally been the domain concerned with societal constraints. Its concerns have been with managing 'aggregate demand' and 'aggregate supply' so as to avoid the crippling depressions of the early 20th century. Post World War II, in the 'golden era' under the influence of Keynes, it appeared that this objective was being achieved.

The consensus was shattered by the sharp decline in growth of the 1970s. In this period the Keynesian models came under attack for not being based on proper 'microfoundations'. The mathematical theories which replaced them all tried to explicitly build up aggregate properties from microeconomic models. A key concept guiding this process was that of 'rational expectations'. The core idea behind this principle is that the market participants are forward looking and will be able to learn how the 'system' functions. So if, for example, the Reserve Bank continually injects money every time there is an economic downturn, people will eventually anticipate this behaviour and act in ways which then undercut the efficacy of that intervention. At a more conceptual level, the principle of 'rational expectations' can be thought of as an internal consistency check: if the model assumes that Y will always follow X, then the agents in the model should also be able to learn that Y will always follow X and act in accordance with that assumption.

One of the consequences of the 'rational expectations' revolution in macroeconomics was increasing scepticism about the effectiveness of policy more generally. If governments knew that doing X was going to cause Y, the rest of society would know this as well and would take steps to anticipate the outcome before X was even fully enacted. In market after market economists became convinced that government intervention was at best ineffective (if agents were free to act so as to undo what the government hoped to achieve) and at worst distortionary.

Some of the models built in adjustment costs which clawed back some role for policy, but on the whole the period since the 1980s saw a roll-back of the interventionism associated with the welfare state. The models that have recently been employed are called 'dynamic stochastic general equilibrium models'. They bolt together microeconomic general equilibrium (with a rational expectations twist) with growth processes (the 'dynamic' part) and random shocks.

The 2008 crash raised serious questions about the relevance of these models. A recent trenchant critique[7] suggested that there were several problems with the way in which these models were set up:

- The models all posit a 'representative consumer', i.e. all individuals are modelled as though they have identical preferences. The reality is that agents are heterogeneous and this heterogeneity matters for the way in which economic processes play out.
- The models assume that markets are perfect, that all agents have access to perfect information and that they can transact with no cost. However, real markets do not work like this. Indeed one of the big changes in microeconomics since the 1970s has been serious attention to the impact of these problems. For instance, in the absence of full information, not everyone who 'deserves' credit will be able to borrow. These problems will be exacerbated if contracts cannot be fully enforced, i.e. if it is not clear that individuals who could repay a loan will, in fact, do so. These constraints imply that many people will not get access to credit. They will thus save or hoard money for a bad day even if it would be theoretically more rational for them to spend it and, indeed, when the government is desperately trying to encourage spending to promote growth.
- The macroeconomic models also neglected the importance of the financial sector and in particular the specific institutional arrangements in different countries for dealing with debt.

Of course the reason why 'representative agent' models were adopted and transaction costs were ignored, is that these models are more tractable. The more heterogeneous the agents and the more imperfect the markets, the more difficult it is to model them – either analytically or even by simulation methods.

This short overview has highlighted several issues. Firstly, the set of mathematical tools available to economists has grown substantially over time. Nevertheless the success in properly describing or predicting behaviour is still far short of where it should be. Secondly, many economic theories have been developed with an eye on 'tractability', i.e. the ease with which they can be solved out, but in the process crucial features of the process to be modelled have been lost. Thirdly, economic theories have a way of 'looping back' into the material that they are supposed to be about. This can happen in two ways. Either agents absorb the theories and adapt their behaviour to align with the theory (e.g. when economic students use game theoretic concepts to work out how to behave in strategic interactions), or agents use the theory to anticipate the behaviour of other actors (e.g. the Reserve Bank) and potentially thereby undercut the outcome predicted by the theory. Modelling these self-referential loops in ways that go beyond the simple 'rational expectations' framework is one of the outstanding questions at the moment.

Finally, economic processes cannot be divorced from how 'institutions' in society operate, whether this be the way in which norms and laws either permit exploitation or foster cooperation, or the nature of the 'credit architecture' that channels borrowing or debts. A consideration of governance and politics and how this is incorporated into economic models is an important issue for theory.

Mathematics in applied research

It is safe to say that although there are lots of sophisticated economic models out there, their success in describing more than simple interactions (e.g. as captured by the 'prisoner's dilemma') is underwhelming. In taking models to 'the world', a number of additional tools derived from mathematics are used.

Econometrics is the branch of economics which marries mathematical models, statistics and economic theory. The workhorses of econometrics are similar to those of statistics – multiple regression analysis and related linear models. Nevertheless, there are some peculiarities to economic applications which have given econometrics a very particular flavour. One of the key issues is that the individuals from whom we collect the data to model behaviour (e.g. to estimate the determinants of wages) are not passive. Indeed, they may have an interest in the outcome of the research which can influence both whether they participate and what sort of information they are willing to volunteer. These 'sample selection' issues have been investigated for a while and econometricians have devised various techniques for dealing with them. Underlying all these are mathematical assumptions about how the response process works. Again these techniques have potential loops: it is difficult to understand who responds and how without reliable data, but the data cannot be 'corrected' to be made reliable without understanding the response process.

Another type of empirical model, used particularly in central banks and national treasuries, is the class of 'computable general equilibrium models'. These models attempt to analyse system-wide interactions. Typically they involve numerous 'sectors' (different industries) that buy from each other and sell output to 'households' and foreign countries, among others. These models are used to predict how particular 'shocks' will propagate through the economy. Unfortunately these models are quite opaque and the constituent equations are often not made available. Some of the equations in the model are estimated, but many of the key parameters are 'calibrated', i.e. they are selected by trial and error to get output that looks plausibly like that from the South African economy. This process is, of course, also not that transparent. The behaviour of the model depends on its structure, on the parameters that are chosen and how it is 'shocked'. The bigger the model the better it should be able to capture actual behaviour, but the more 'degrees of freedom' there are in tweaking parameters to get the desired output. It is not clear that the resulting predictions are worth the paper they are written on. The main function of such models seems to be to shut down debate about policy options rather than honestly explore the possibilities and the uncertainties associated with different choices.

Mathematics in teaching

Given the importance of models to much of economic theory and research, it stands to reason that trainee economists should be exposed to the mathematical techniques involved. Accordingly, economics departments often have a maths entrance requirement. In the case of the University of Cape Town, this requirement is actually quite high. Nevertheless, students struggle with the abstract nature of the models and as a result economics has in most standard undergraduate treatments replaced the mathematical relationships with graphical techniques. Consequently, first-year textbooks introduce many 'curves' (e.g. supply and demand or 'indifference curves') and then spend a lot of time discussing how these curves shift with changes in the underlying parameters of the model.

Students spend so much time trying to absorb the technical details that they often lose sight of the broader logical structure underpinning the model. Instead of understanding the model as a hypothetical construct (i.e. if the axioms of choice are true then we would expect to see the following behaviour), they view it as a set of statements of a priori truths, i.e. the models become quasi-theological statements which are impervious to disproof. This could be remedied to some extent if students saw alternative models earlier in their careers. There are now attempts under way to construct new syllabi that introduce transaction costs and non-clearing markets earlier in the sequence. Unfortunately some of these models make even higher mathematical demands.

And increasing the mathematical rigour of our teaching also imposes costs. For many students mathematics is seen as a straightforward barrier to entry. Economics and commerce courses are in big demand because they seem to offer a reliable route to higher paying jobs. Given the pressures to expand access, increasing the mathematical content may very well be seen as a retrograde step.

Mathematics in South African economics

Indeed at present it is not clear that for many South African economists the level of rigour is even strictly required. Very few South African economists work at the cutting edge of either economic theory or the application of new mathematical and statistical techniques to applied problems.

Nevertheless there are many opportunities for tackling some of the 'big' issues in South Africa. Modern economic theory has focused on the interplay between 'institutions' and economic processes. South Africa is arguably one of the better 'laboratories' in which to think about these connections. The history of apartheid and the process of transformation since 1994 provides a backdrop against which many of these issues can be researched, both empirically and theoretically. Of course a better modelling of these processes would undoubtedly loop back into the policy discussion itself. Getting the right concepts and tools for dealing with those questions could be enormously productive.

References

1. Leijonhufvud A. Life among the Econ. Econ Inq. 1973;11(3):327–337. https://doi.org/10.1111/j.1465-7295.1973.tb01065.x
2. Von Neumann J, Morgenstern O. Theory of games and economic behaviour. 3rd ed. Princeton: Princeton University Press; 1953.
3. Young HP. Individual strategy and social structure: An evolutionary theory of institutions. Princeton: Princeton University Press; 2001.
4. Acemoglu D, Robinson JA. Why nations fail: The origins of power, prosperity and poverty. London: Profile Books; 2012.
5. Kahneman D. Thinking fast and slow. New York: Farrar, Straus and Giroux; 2011.
6. Henrich J. The secret of our success. Princeton, NJ: Princeton University Press; 2016. https://doi.org/10.1515/9781400873296
7. Muellbauer J. Macroeconomics and consumption. Working Paper 811. Oxford: Economics Department, Oxford University; 2016. Available from: https://ideas.repec.org/p/oxf/wpaper/paper-811.html