Finding synergies between the mathematical and physical sciences

AUTHOR:
Daya Reddy[1]

AFFILIATION:
[1]Department of Mathematics and Applied Mathematics, University of Cape Town, Cape Town, South Africa

CORRESPONDENCE TO:
Daya Reddy

EMAIL:
daya.reddy@uct.ac.za

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Depending on one's sources, the mathematical historical record dates back to around 1200 BCE.[1,2] We are more familiar in the West with the seminal contributions of ancient Greece, although the mathematical heritage of the Islamic world from the 8th century onwards has become increasingly better known. Nevertheless, one finds, in many different parts of the world, engagement of a mathematical nature dating from antiquity. A further example relates to the Mayan civilization of central America: evidence of mathematical activity there appears to date back at least to 50 BCE. Similar remarks may be made in respect of mathematical developments in sub-Saharan Africa.[3]

While much is known, there is uncertainty about attributions of some mathematical concepts, and also of the extent to which communities in different parts of the world communicated with each other on topics of a mathematical nature. Certainly the picture that emerges is one of developments of particular concepts in multiple locations – sometimes contemporaneous, often at different times – with ebbs and flows in creativity, yet also with examples of cross-pollination. Thus the picture of a society or region operating in complete isolation, laying claim to specific inventions and developments, and with these ideas possibly flowing, if at all, in one direction, is at odds with the admittedly incomplete record. An example would be the complicated history of the concept of zero, its symbolic representation, and its possibly multiple origins in various parts of Asia, as well as in the Mayan civilization.[1]

A striking and enduring feature of the development of mathematics is the central role played by the physical sciences, most notably astronomy, in motivating mathematical developments. Engineering also has played a significant role, spurred for example by military considerations.

The late 17th century onwards witnessed a great flowering in mathematics, inspired by the quest for knowledge in the physical sciences, with central figures including Isaac Newton, Gottfried Wilhelm Leibniz and Leonhard Euler on topics in mechanics and astronomy, the studies of Joseph Fourier on heat conduction, and of James Clerk Maxwell on electromagnetism. These topics in turn paved the way in the 20th century for the paradigm-shifting theories of relativity and quantum mechanics.

The intense relationship between mathematics and the physical sciences has not abated at all with the passage of the 20th century. Examples of groundbreaking achievements in physics that are underpinned by innovative mathematics include the work of astrophysicist Subramanyan Chandrasekhar, who won recognition, including a Nobel Prize in Physics in 1983, for his formulation of theories for the stability and evolution of stars, including those that subsequently undergo collapse into compact brilliant stars known as white dwarfs.

The second half of the 20th century has seen the emergence and maturation, in parallel with the advent of computers of ever-increasing power, of what is referred to as computational science, or scientific computing: that is, investigations in which advanced computing capabilities are used to understand and solve complex problems in the physical and biological sciences, engineering, social sciences, and a myriad of other areas. The essence of computational science is the development of algorithms based on mathematical models, turning these into computer code and other forms of software, and their use in simulation. It is now commonly accepted as a peer methodology alongside the traditional forms of investigation, namely experiment and theory. It is appropriate in considering the beginnings of scientific computing to mention the remarkable work of Ada Lovelace (1815–1852), among whose most important contributions was the first published description of a stepwise sequence of operations for solving certain mathematical problems. Her contributions included the visionary idea of a machine that could manipulate symbols in accordance with a set of rules, rather than simply calculate.

Mathematics remains as central to the physical sciences and engineering as ever. It provides the language and the avenues through which to develop models of physical phenomena which provide insights into the phenomena concerned, as well as predictive capabilities that may, for example, suggest particular directions in experimental studies. In the world of engineering such models are central to design – that is, the development of devices or structures to perform a specific function. Mathematical modelling furthermore is used in the optimal design of processes and components through procedures that allow for sequential improvement of designs, each time tweaking the previous attempt in a systematic way by modifying parameters such as the geometry or material composition.

But the relationship between mathematics and the physical sciences and engineering is much more than that. It is a symbiotic one, in the sense that problems in the physical sciences provide fertile ground for the development of new mathematical theories and techniques. The late 20th century has been witness to particularly striking examples of areas of physics such as quantum field theory giving rise to significant new insights in mathematics, and to remarkable and deep mathematical theories.[4] An acknowledgement of the two-way nature of these relationships is crucial to guiding the way in which we shape curricula, and design and pursue research programmes.

The interwoven nature of mathematics with the physical sciences is illustrated here through a description of the 'life cycle' of a problem arising in mechanics and materials science, and concerned with the behaviour of metals under various loading conditions. A researcher seeking to explore this problem from a mathematical perspective would begin by developing a model that captures in a mathematical sense the properties of the material. By combining these properties with well-established laws of physics such as conservation of mass and momentum, one obtains what is known as a system of partial differential equations, that is, a set of equations that describes the variation of quantities in position and time. These equations constitute the mathematical model in this instance.

The next step in the process is naturally that of seeking a solution; with this at our disposal we would be able to use the model to explore its predictive capabilities, and if relevant, engage in the process of design to meet a specified need. Here we inevitably hit an obstacle, for it is seldom the case that realistic models are amenable to exact solution. What to do? As a first step, the mathematical process continues by finding out as much as possible about the solution, even without being able to construct it. This is known as qualitative analysis, and is an extremely important part of the investigation.

The qualitative process then sets the scene for the next best thing, of finding an approximate solution: one that is not exact, but which is sufficiently close to meet our needs. It is at this point that the computer comes into play: the investigator develops techniques and associated algorithms, that is, a logical sequence of steps and calculations, which are translated into code, and through which the approximate solutions are generated. An important part of this step is validation: is the model a sufficiently reliable representation of the phenomenon? Then, we must undertake the process of verification: has the problem led to an approximation of sufficient accuracy? And, are we able to quantify the error in the approximation? The two processes of validation and verification are sometimes likened to asking (1) whether we have solved the right problem and (2) whether we have solved the problem right! Once these questions have been answered satisfactorily, the process of exploration or design can then begin in earnest.

The above description addresses through a simple example one direction in the two-way relationship between mathematics and the physical sciences. What about the other?

Well, it may be that the process of exploration yields phenomena that had not been anticipated, but had perhaps been observed in simulations, and which were therefore not accounted for in the qualitative analysis. For example, under certain conditions the metal sample may undergo deformation in which parts of its bulk slide internally relative to each other. It is precisely these kinds of phenomena that led, more than three decades ago, to the development of a rich mathematical theory of function spaces which are able to capture such behaviour. This was new mathematics, whose genesis was the study of the physical phenomenon of plastic or irreversible deformation.

This example illustrates the highly interdependent nature of mathematics and the physical sciences. It also addresses an apparent dichotomy, between pure and applied mathematics, that has bedevilled mathematics particularly since the 20th century. Such a distinction has been largely absent in the development and practice of mathematics, from antiquity through to the early 19th century: typically mathematicians would move effortlessly back and forth between what we today refer to as pure and applied mathematics. While the dichotomy has been given credence, particularly in the mid- to later 20th century, there is now a growing acceptance that it is a counterproductive, if at all valid, dichotomy.

Rather, a more appropriate metaphor is the description by American mathematician Robert Zimmer of mathematics as a fabric – a woven artefact that derives its strength from its interconnectedness, but which may be weakened by a tear or gap.[4] Another view of the corpus of mathematics is that of a grand edifice, a cathedral, with its practitioners working alone or in small groups on creating beautiful and functional components, and who would, when stepping back, be able to admire the totality of the creation.

Likewise, the organisational distinction between statistics and mathematics is unproductive and can hinder interdisciplinary work. In developing mathematical models, the aim is always to develop a model that is sufficiently realistic, yet not so complex as to be intractable, even computationally. There is an element of uncertainty in most physical processes and phenomena, which is not captured by purely deterministic models. In this context, uncertainty quantification has become a major area of endeavour, and one in which there is a healthy interaction between mathematicians, statisticians and probabilists.

While the physical sciences provide the example par excellence of a set of disciplines that have a hugely fertile relationship with mathematics, they are not unique in this regard. Indeed, similar relationships exist between mathematics and areas such as chemistry, biochemistry, molecular biology, demography and other social sciences. Economics is another example: consider the mathematicians who have been awarded the Nobel Memorial Prize in Economic Sciences!

As with any endeavour that crosses disciplinary boundaries, the key challenge, over and above that of defining and formulating the problem, is one of navigating these boundaries in such a way as to develop an intimate understanding of other disciplinary 'cultures': ways of communicating, methodologies, and of understanding what is important. Such cross-pollination requires dedicated time and energy, and the obstacles can sometimes be frustrating, but with patience it is a way of working that brings rich rewards.

As has always been the case, new times bring new opportunities. In this regard, the dramatic technological advances of the 20th century have played a major role in opening up new areas of enquiry. Mathematics, like all other disciplines, must be agile enough to be able to respond energetically to these new opportunities.

An example is the emergence of data science as a product of the digital revolution – a reference to the unprecedented explosion in the capacity to acquire, store, manipulate and transmit huge volumes of data. The emergence of big data, as it is referred to, opens the way to new and exciting challenges across many disciplines. These scientific opportunities lie in identifying and characterising previously unseen patterns and unsuspected relationships, being able to simulate highly complex system dynamics, and mapping complex states. Whatever the opportunities – in studying environmental change, climate forecasting or migration patterns, for example – these will require multidisciplinary approaches, and mathematics will necessarily occupy a central place in these collaborations. It is vital that our students and researchers be ready to grasp these opportunities.

What are the implications of these developments for curricula in mathematics? Continuous reflection is required around the question as to the characterisation of a well-rounded mathematics graduate in the modern era. It is essential that students be exposed not only to the manner in which mathematics is applied to other areas, but also to the ways in which its own development is influenced by progress in areas outside mathematics. Thus, the interactions between mathematics and other areas in the natural sciences and beyond, and the two-way nature of these interactions, should inform curriculum development in a direct way.

These considerations may lead to new courses, majors and partnerships with other disciplines. Computation, which can aid discovery and provide new insights as well as serve as a functional tool, should be an integral part of mathematics curricula. Likewise, the place of topics such as uncertainty quantification and its foundational elements in randomness and probability should be carefully evaluated.

A certain degree of flexibility is required so that mathematicians are able to discern new directions and opportunities, and are willing to ensure that these developments influence the structure and content of curricula. The fundamental attributes of mathematics such as conceptual and abstract thinking and deductive reasoning will always be present and represent essential skills for any number of careers. Equally important is the need to convey, whether through teaching at all levels or in sharing research ideas, the inherent beauty of mathematics and its cultural value.

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