



Cosmic ray propagation in a fractal galactic medium

Authors:

Hamid A. Kermani¹
Jalileldin Fatemi¹

Affiliation:

¹Department of Physics,
Faculty of Science,
University of Kerman,
Islamic Republic of Iran

Correspondence to:

Hamid Kermani

email:

ham_arjomand@yahoo.
com

Postal address:

Department of Physics,
Faculty of Science,
University of Kerman,
22 Bahman Avenue,
Kerman, Islamic Republic
of Iran

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Simple cosmic ray diffusion in magnetic fields is often discussed in terms of a characteristic scattering mean free path or equivalent diffusion coefficient. This assumes very simple properties of the structure of astrophysical magnetic fields. A better approximation is to assume that the magnetic structure has fractal properties and there is then the possibility of including very short and very long interaction lengths when modelling the propagation. Results of modelling such propagation in a fractal medium are discussed. Values of the propagation parameter (α) less than 2 were obtained and confirm the plausibility of the hypothesis that supernova are the origin of galactic cosmic rays in the energy range below the knee in the spectrum.

Introduction

It is thought that supernova remnants are the most probable origins of galactic cosmic rays below 0.1 PeV (10^{14} eV)¹ and that these are possible sources of energies up to 100 PeV, above which extragalactic sources are believed to dominate. Even if only 10% of the supernova explosion energy (10^{51} erg) goes into cosmic rays, they are energetically capable of delivering the requisite cosmic ray power of 10^{42} erg/s into the interstellar medium. They may well be the only class of object within our galaxy with this capability. The acceleration mechanism of cosmic ray particles is most probably diffusive shock acceleration, which can produce the appropriate power and satisfy the requirement of a power law energy spectrum.

However, supernova acceleration models, such as those of Axford², Berezhko et al.³ and others, confront many problems. For example, none of the existing supernova models can demonstrably provide the necessary acceleration efficiency up to the observed energies (greater than 1 PeV) of galactic cosmic rays. Also, in such models, the total energy of particles extracted from the shock energy is found to be greater than 10%. This is much higher than required to provide the observed energy which is needed to fit the observed flux at Earth using the conventional galactic propagation models.

The problems with a supernova model can be approached in two ways. The first is to look for modifications to the model for the origin of the galactic cosmic rays. The second is to look for a different model for the propagation of particles in the interstellar medium of the galaxy, such that the relationship between the observed energy density and the total source energies is changed.

In this paper, we discuss the second approach and examine whether particle density as a function of the galactic radius has the required features within a model for the diffusion of cosmic rays in an interstellar medium with fractal structure.

It is known that the distribution of matter and magnetic fields in the galaxy is highly non-homogeneous on different spatial scales. Gaseous clouds with very different densities, temperatures and degrees of ionisation move through space in a highly turbulent way. Galactic structures such as shells, clouds and filaments are widely spread in the interstellar medium. We can say that the galaxy has a multi-scale structure or even a multi-scale length structure.⁴ Therefore we wish to consider the consequences of cosmic ray propagation in a multi-scale turbulent medium.

Cosmic ray propagation has usually been assumed to be in a form of normal diffusion, resulting in a Gaussian spatial distribution of particles around a source. We will now assume that, (1) cosmic rays with energies below the knee of the cosmic ray spectrum at 3 PeV ($1 \text{ PeV} = 10^{15}$ eV) are of supernova origin and that, (2) based on a recent suggestion by Lagutin and colleagues⁵, the propagation of such particles in an interstellar medium with a fractal structure is described by a different diffusion equation to that usually assumed.

The result we obtained differs from those usually found, because the cosmic rays now propagate in a non-homogeneous interstellar medium.



Super-diffusion

On the basis of the assumption that the interstellar medium has a fractal structure, Lagutin et al.⁵ have formulated and solved analytically the equation for anomalous diffusion in a fractal medium for different input conditions. The super-diffusion equation formulated in Lagutin's work has the general form:

$$\frac{\partial N(r, E, t)}{\partial t} = -D(E, \alpha)(-\Delta)^{\alpha/2} N(r, E, t) + S(r, E, t) \quad [\text{Eqn 1}]$$

Where $N(r, E, t)$ is the number of cosmic ray particles with the energy of E at the distance r from the source, D is the anomalous diffusion coefficient, $(-\Delta)^{\alpha/2}$ is the fractional Laplacian called the 'Riss' operator and $S(r, E, t)$ is the function describing the density distribution of sources.

To obtain the number of particles at a distance r from a supernova (assumed to be a point source with an inverse power spectrum), we followed the suggestion of Lagutin et al.⁵, and used the steady-state diffusion equation:

$$D(-\Delta)^{\alpha/2} N(r, E) = S(r, E) \quad [\text{Eqn 2}]$$

Finally, we find the solution of the steady-state case of the diffusion equation as:

$$N(r, E) = \frac{2^{-\alpha} S_0 \Gamma(\frac{3-\alpha}{2})}{\pi^{\frac{3}{2}} D_0 r^{3-\alpha} \Gamma(\frac{\alpha}{2})} E^{-p-\delta} \quad [\text{Eqn 3}]$$

Where E^{-p} is the energy spectrum of source (p is related to the effect of source) and $\alpha = 2\delta$ is the intrinsic property of the interstellar medium. The details of the solutions to [Eqn 1] and [Eqn 2] are given by Lagutin et al.^{5,6}

Lagutin et al.⁵ suggested the investigation of such a super-propagation regime and calculated the propagation parameter (α) range, specific to such a medium $0.6 < \alpha < 2$. In this work, it is assumed that all cosmic rays in the mentioned energy range, are of supernova origin. The spatial distribution of supernovae is given by [Eqn 4]:

$$F(R, Z) = \left(\frac{R}{R_0}\right)^a \exp\left(-b\left(\frac{R}{R_0} - 1\right) - \frac{Z}{Hz}\right) \quad [\text{Eqn 4}]$$

Actually, the intensity of cosmic rays (I), in every energy range is proportional to $N(r, E) F(R, Z)$. The intensity in terms of R for different α , is shown in Figure 1.

Radial distribution of supernovae in the galactic disk with cylindrical symmetry is calculated using [Eqn 4]. Where $a = 1.69 \pm 0.22$, $b = 3.33 \pm 0.37$, $R_0 = 8.5$ kpc (distance of the sun from the galactic centre), $Hz = 0.2$ kpc² (the vertical scale parameter of the galaxy) and Z is the vertical distance from the galactic disk. And we know that the producing rate of supernovae (Type II) in the galaxy is 10^{-2} per year.

We present the results of a Monte Carlo simulation using a diffusion equation appropriate to a non-homogeneous medium. As we noted above, the assumption of normal

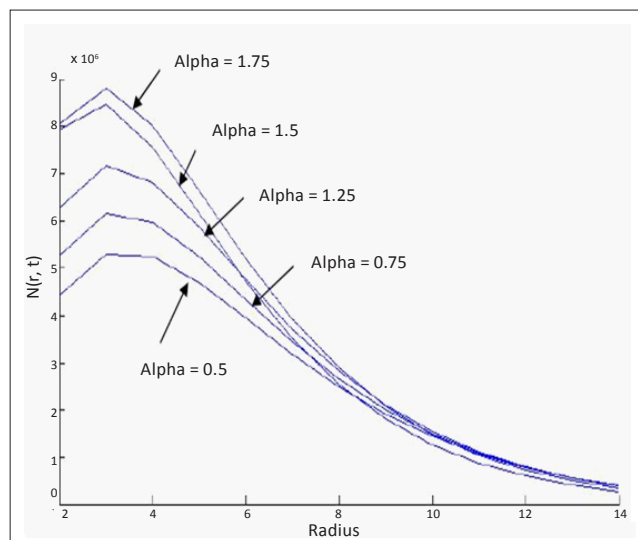


FIGURE 1: Intensity of cosmic rays produced by supernovae in the fractal interstellar medium for different values of alpha, α .

diffusion, resulting in a Gaussian spatial distribution of particles, does not easily reproduce observed cosmic ray fluxes. We will now examine the suggestion by Lagutin et al.⁵ and consider an interstellar medium with scales described by a fractal structure, resulting in an 'anomalous' diffusion equation.

By assuming a Kolmogorov-type spectrum for the galactic magnetic field strength, a trajectory and relative galactic containment times of cosmic rays for conventional and super-diffusion propagations are simulated.

Parameterising the diffusion

The parameter α , given in Lagutin et al.^{5,6}, is the basic parameter for describing the propagation of cosmic rays in an anomalous interstellar medium. Its magnitude is related to the spectrum of magnetic irregularities of the medium. The standard diffusion model (for particles in a homogenous turbulent medium), which leads to the Gaussian distribution of particle densities, is found when $\alpha = 2$. On the other hand, $\alpha < 2$ is related to the anomalous super-diffusion regime.^{5,6} We will see the detailed role of parameter α below. Our results will show the usefulness of assuming that the interstellar medium has a fractal structure with $\alpha < 2$.⁵ This assumption, which is consistent with our physical picture of galactic magnetic fields, will reduce the discrepancy between expectation and observation for cosmic ray fluxes in supernova models, noted above, through changes to the galactic propagation characteristics of cosmic rays originating in supernovae.

Super-diffusion and galactic containment times

Because the interstellar medium has a multi-phase character and is non-homogenous, the turbulence level is high and the ratio of the mean amplitude irregular magnetic field in the galaxy determined by the turbulence of the regular field undoubtedly varies from place to place, depending on the

proximity of stars of various types and shocks of a wide range of strengths. The way to advance on the simple homogenous diffusion approximation is by using the so-called anomalous diffusion scenario.⁸ Here there are two regimes, (1) sub-diffusion, in which there are relatively small spatial displacements and (2) super-diffusion, where there are large displacements (the so-called levy flights). For a random walk leading to super-diffusion (levy flights), the step length l is chosen from a probability distribution that decays as $P(l) = l^{-\mu}$ for large l , where $\mu < 3$; for normal diffusion, the step-size distribution has a decay exponent of $\mu > 3$.

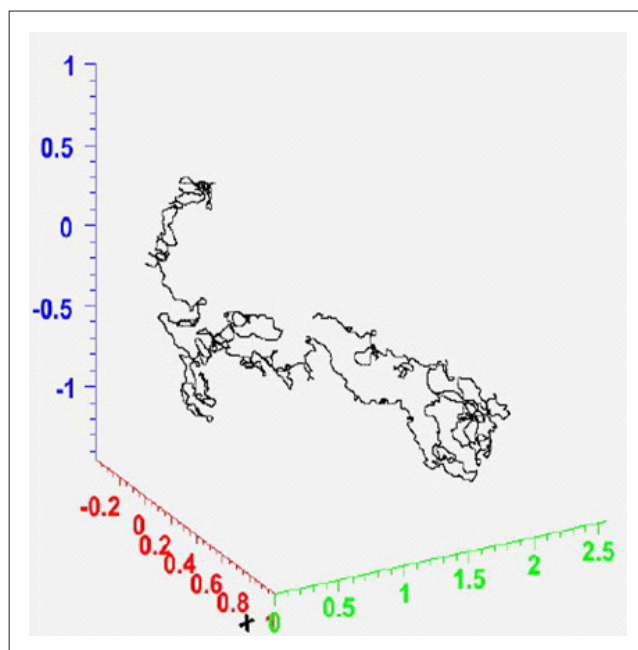
Super-diffusion has the effect of applying a diffusion mean free path which is variable within the limits set by its being drawn from a fixed statistical distribution. Unlike more conventional diffusion, this has the effect of occasionally allowing the diffusing particle to travel an unexpectedly large distance between interactions. On the other hand, there may also be many more short distances between interactions (Figure 2).

When considering the containment time for cosmic ray particles of the same composition within a galactic magnetic field, it is the former effect which is most important. A cosmic ray will occasionally travel a large distance between interactions and this may be sufficient for it to leave the galactic magnetic field unexpectedly quickly. The effect of this process is to reduce the overall containment time (and hence the predicted energy density of cosmic rays) at energies where containment is usually effective, that is, the range of energies between the knee and the ankle of the cosmic ray energy spectrum. This is shown in Figure 3, in which calculations for cosmic ray propagation in turbulent galactic magnetic fields give a greater galactic residence time for conventionally diffusing particles compared to super-diffusion particles at energies below about 100 PeV (10^{17} eV).

Super-diffusion to derive an appropriate galactic cosmic ray energy density

We modelled the super-diffusive propagation of cosmic rays in galactic magnetic fields. We assume that supernovae are the nuclei cosmic ray sources, but for cosmic ray electrons other sources with different acceleration efficiency also can contribute.⁹ In the case of supernovae, they are distributed within the galactic plane with a scale height from the plane of 0.2 kpc. We assume a supernova radial distribution that is normalised to one supernova in the galaxy, as is shown in Figure 4.

We calculated the energy density of cosmic rays from a single supernova and then extended this to supernovae distributed throughout the galactic volume. This allowed us to determine a radial gradient of the galactic cosmic ray density and a local value for the cosmic ray energy density. Figure 5 shows the radial gradient results and the spread in acceptable values of α to be derived from the gradient (shown in Figure 5 as



Note: Distance scales are in units of kpc.

FIGURE 2: A cosmic ray trajectory in a $1 \mu\text{G}$ turbulent magnetic field as might be found in our galaxy. This track is modelled by following a 10 PeV proton through Kolmogorov turbulence (equivalent to a conventional diffusion model). Super-diffusion models have occasional long straighter paths between major changes of direction.

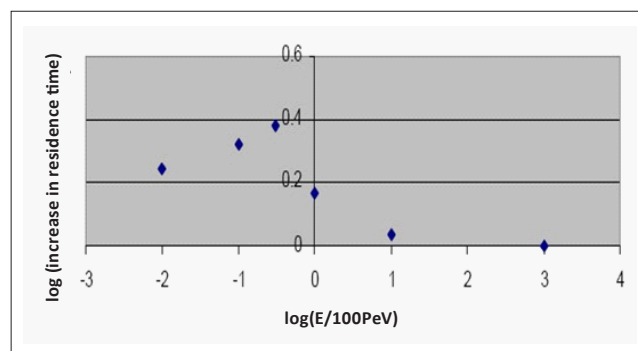


FIGURE 3: Relative galactic cosmic ray containment times for conventional and super-diffusion propagation.

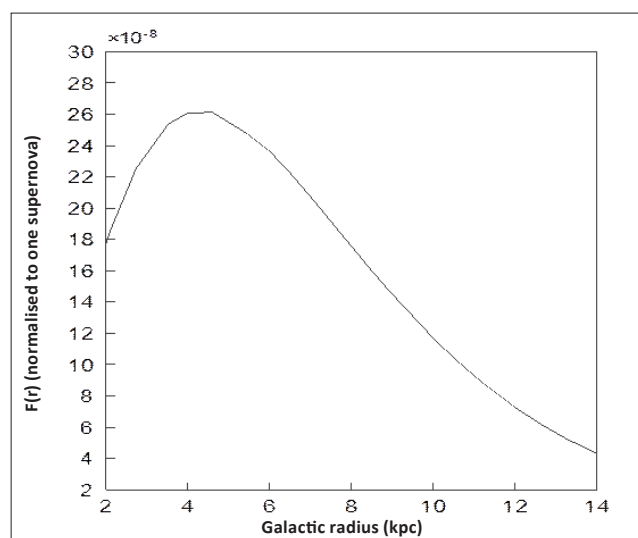
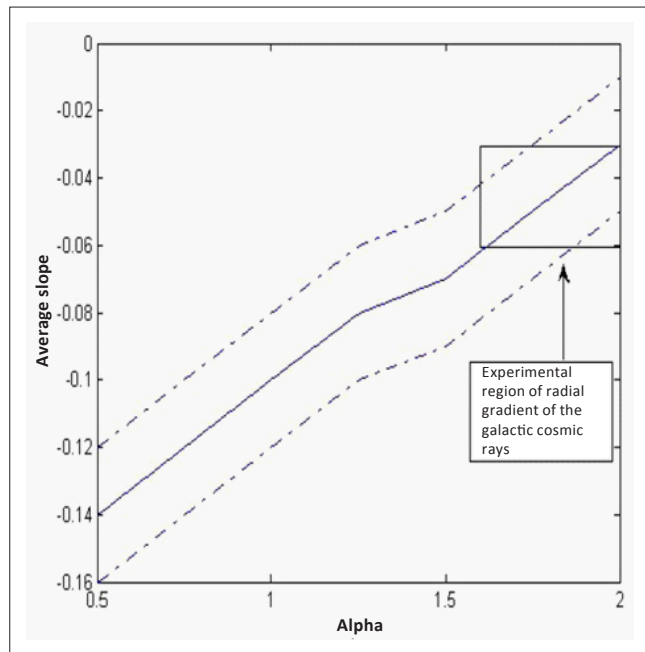


FIGURE 4: Assumed radial distribution of supernovae normalised to one supernova in the galaxy.



Note: The experimentally observed range is indicated by the box.

FIGURE 5: Modelling results showing the calculated relationship between the cosmic ray radial gradient (fraction/kpc) in the galaxy and the propagation parameter α .

the solid line). Also using the standard deviation method, we find that for every α the error is 0.02 (shown in Figure 5 as the dashed lines above and below the full line). Furthermore, the experimental region of the radial gradient range, between 0.03 kpc^{-1} and 0.06 kpc^{-1} ,^{10,11} is consistent with our result.

Assuming a Type II supernova rate of 1/100 per year, we can use super-diffusion ideas to calculate the cosmic ray energy density distribution throughout the galactic disk. Such modelling has the super-diffusion parameter α as a free parameter (remembering that $\alpha = 2$ gives conventional diffusion). We can then use the known cosmic ray spectrum and radial gradient in the vicinity of the solar system to define an energy density at that radial distance for comparison with the modelling results. The result of this comparison is that the best fit for the value of α is about 1.65. Possible fits range from 1.6 – 1.9, but no acceptable fit is found for $\alpha = 2$, which would correspond to conventional diffusion. This confirms our original expectation that conventional diffusion is unable to fit the measured cosmic ray energy density at the galactic radial distance of the solar system.

Summary

Our calculations show that the interstellar medium within which galactic cosmic rays produced by supernovae propagate, may have a fractal and non-homogenous structure. If our results are compared with the experimental values of the radial gradient of cosmic rays (in the range of $0.03 \text{ kpc}^{-1} - 0.06 \text{ kpc}^{-1}$)^{10,11} and also the simulated values of energy density of cosmic rays with the expected ones ($1.8 \times 10^{-4} \text{ eV/cm}^3$), values of α less than 2 are obtained for the super-diffusion equation suggested by Lagutin et al⁵. This does not fit the normal diffusion of particles in a homogenous medium with a Gaussian spatial distribution, which would result if $\alpha = 2$. As a result, this work confirms the plausibility of the hypothesis that supernovae are the likely origin for galactic cosmic rays in the energy range below the knee in the spectrum ($\sim 1 \text{ PeV}$), providing that the propagation of particles is in a fractal interstellar medium ($\alpha < 2$).

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